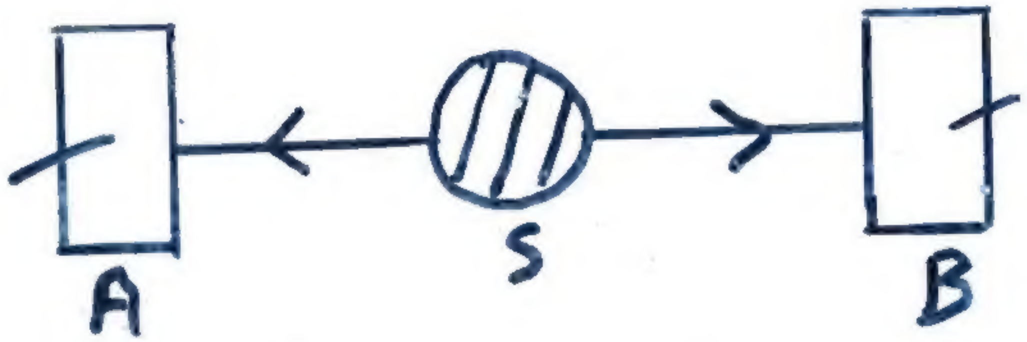
RELATIVITY AND QUANTUM MECHANICS:
CONFLICT OR PEACEFUL COEXISTENCE

New York January 1986

STOCHASTIC HIDDEN-VARIABLE THEORIES

Measure $\underline{a} = \underset{\sim}{\sigma}_A \cdot \hat{\underset{\sim}{a}}$



Measure $\underline{b} = \underset{\sim}{\sigma}_B \cdot \hat{\underset{\sim}{b}}$

Assume existence of triple joint $\text{Prob}_{\underline{a},\, \underline{b},\, \underline{\lambda}}(\varepsilon_a, \varepsilon_b, \varepsilon_\lambda)$

Jarrett Completeness

$$\text{Prob.}(\varepsilon_a \mid \varepsilon_b \,\&\, \varepsilon_\lambda) = \text{Prob}(\varepsilon_a \mid \varepsilon_\lambda)$$

$$\mathrm{Prob}^{|\psi\rangle}(\underline{a} = \varepsilon_a \ \& \ \underline{b} = \varepsilon_b \ \& \ \underline{\lambda} = \varepsilon_\lambda)$$

$$= \mathrm{Prob}(\underline{a} = \varepsilon_a \mid \underline{b} = \varepsilon_b \ \& \ \underline{\lambda} = \varepsilon_\lambda)$$

$$\times \ \mathrm{Prob}(\underline{b} = \varepsilon_b \mid \underline{\lambda} = \varepsilon_\lambda)$$

$$\times \ \mathrm{Prob}^{|\psi\rangle}(\underline{\lambda} = \varepsilon_\lambda)$$

<u>The Jarrett Condition</u>

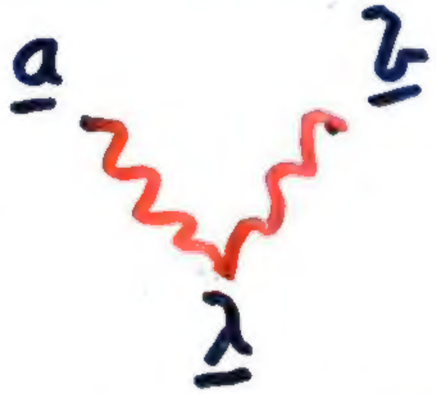
$$\mathrm{Prob}(\underline{a} = \varepsilon_a \mid \underline{b} = \varepsilon_b \ \& \ \underline{\lambda} = \varepsilon_\lambda)$$

$$= \mathrm{Prob}(\underline{a} = \varepsilon_a \mid \underline{\lambda} = \varepsilon_\lambda)$$

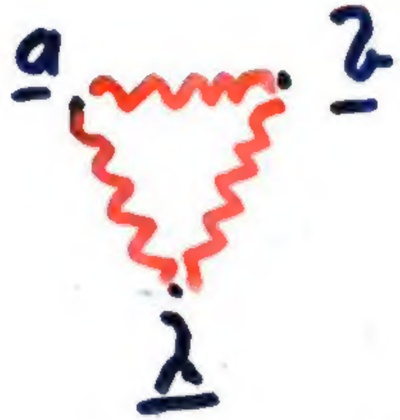
$\Rightarrow$ FACTORIZABILITY $\Rightarrow$ BELL INEQUALITY

HOW TO EXPLAIN CORRELATIONS BETWEEN a and b

(1) Common Cause



(2) Combination of common cause and direct cause



(3) Direct Cause

(4) Passion

3

NECESSARY CONDITION FOR STOCHASTIC CAUSALITY

$$\underline{a} \rightsquigarrow \underline{b} \xleftarrow{\quad} \underline{d}$$

$\underline{b}$ screens off $\underline{a}$ from disturbance $\underline{d}$

i.e. $\exists D \left(\forall \underline{d} \in D \left(\mathrm{Prob}(\underline{a} = \varepsilon_a \mid \underline{b} = \varepsilon_b \,\&\, \underline{d}) = \mathrm{Prob}(\underline{a} = \varepsilon_a \mid \underline{b} = \varepsilon_b) \right) \right)$

PERTURBATIONS OF THE SINGLET STATE

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\sigma_{Az}=+1\rangle|\sigma_{Bz}=-1\rangle - |\sigma_{Az}=-1\rangle|\sigma_{Bz}=+1\rangle\Big)$$

We require

$$\exists D\ \forall \underline{a}\ \forall \underline{b}\ \Big(\forall |\psi'\rangle \in D$$

$$\Big(\mathrm{Prob}^{|\psi'\rangle}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b)$$

$$= \mathrm{Prob}^{|\psi\rangle}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b)\Big)\Big)$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}}\Big(|\sigma_{Az}=+1\rangle\, U(\hat{\underset{\sim}{n}},\phi)|\sigma_{Bz}=-1\rangle$$

$$- |\sigma_{Az}=-1\rangle\, U(\hat{\underset{\sim}{n}},\phi)|\sigma_{Bz}=+1\rangle\Big)$$

where $U(\hat{\underset{\sim}{n}},\phi) = e^{i(\underset{\sim}{\sigma}_B\cdot\hat{\underset{\sim}{n}})\phi/2}$

is a general element of $SU(2)$

We have that

$\mathrm{Prob}^{|\psi\rangle}(\underset{\sim}{a}=1) = 1/2$

$\mathrm{Prob}^{|\psi\rangle}(\underset{\sim}{b}=1) = \frac{1}{2}$

$\mathrm{Prob}^{|\psi\rangle}(\underset{\sim}{a}=1|\underset{\sim}{b}=1) = \sin^2\frac{1}{2}\theta_{ab}$

where θ_{ab} is $\angle\, \hat{\underset{\sim}{a}}, \hat{\underset{\sim}{b}}$.

• TRANSFORMATION OF OPERATORS

$$U(\hat{\underset{\sim}{n}},\phi)\,\underset{\sim}{\sigma}_B\,U(\hat{\underset{\sim}{n}},-\phi)$$

$$= R(\hat{\underset{\sim}{n}},\phi)\,\underset{\sim}{\sigma}_B$$

So under unitary transformation induced by $U(\hat{\underset{\sim}{n}},-\phi)$

$$\underline{a} = \underset{\sim}{\sigma}_A \cdot \hat{\underset{\sim}{a}} \rightarrow \underline{a}' = \underline{a}$$

$$\underline{b} = \underset{\sim}{\sigma}_B \cdot \hat{\underset{\sim}{b}} \rightarrow \underline{b}' = \left(U(\hat{\underset{\sim}{n}},-\phi)\,\underset{\sim}{\sigma}_B\,U(\hat{\underset{\sim}{n}},\phi)\right)\cdot\hat{\underset{\sim}{b}}$$

$$= \left(R(\hat{\underset{\sim}{n}},-\phi)\,\underset{\sim}{\sigma}_B\right)\cdot\hat{\underset{\sim}{b}}$$

$$= \underset{\sim}{\sigma}_B \cdot \hat{\underset{\sim}{b}}', \quad \text{where } \hat{\underset{\sim}{b}}' = R(\hat{\underset{\sim}{n}},\phi)\,\hat{\underset{\sim}{b}}$$

<u>Hence</u>

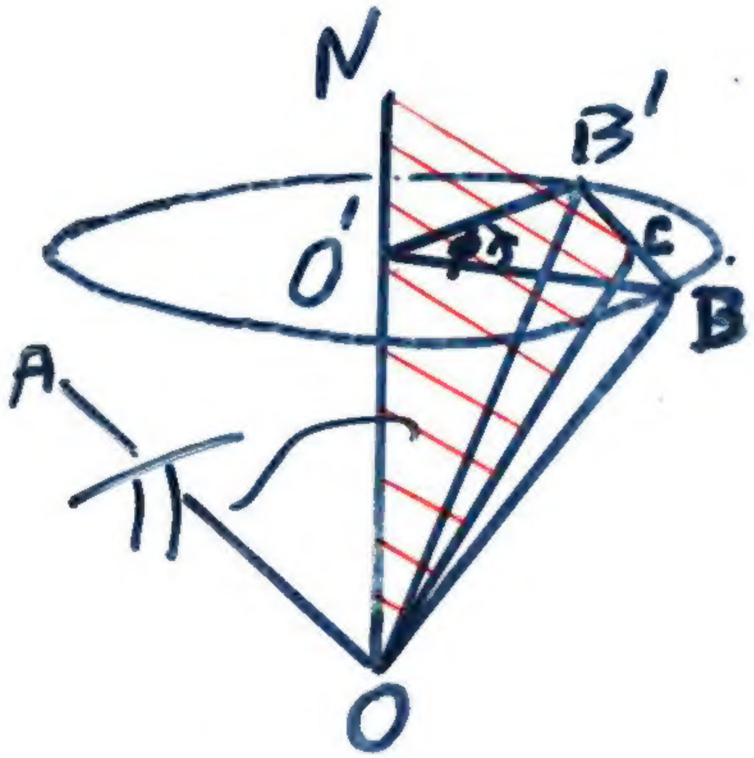
$$\text{Prob}^{|\psi'\rangle}(\underline{a}=1) = \text{Prob}^{|\psi\rangle}(\underline{a}'=1)$$
$$= 1/2$$

$$\text{Prob}^{|\psi'\rangle}(\underline{b}=1) = \text{Prob}^{|\psi\rangle}(\underline{b}'=1)$$
$$= 1/2$$

$$\text{Prob}^{|\psi'\rangle}(\underline{a}=1\,|\,\underline{b}=1) = \text{Prob}^{|\psi\rangle}(\underline{a}'=1\,|\,\underline{b}'=1)$$
$$= \sin^2\tfrac{1}{2}\theta_{ab'}$$

NECESSAY CONDITION, FOR STOCHASTIC CAUSALITY FOR TWO SPIN-½ SYSTEMS

$$\theta_{ab} = \theta_{ab'}$$



$ON = \hat{n}$

$OB = \hat{b}$

$OB' = \hat{b}'$

$OA = \hat{a}$

SIGNALLING AND ROBUSTNESS

$$\text{Prob}(\underline{a} = \varepsilon_a)$$

$$= \sum_{\varepsilon_z} \text{Prob}(\underline{a} = \varepsilon_a / \underline{z} = \varepsilon_z) \times \text{Prob}(\underline{z} = \varepsilon_z)$$

For deterministic case

write $\text{Prob}(\underline{z} = \varepsilon_z) = \delta(z, \varepsilon_z)$

and $\text{Prob}(\underline{a} = \varepsilon_a / \underline{z} = \varepsilon_z) = \delta(\varepsilon_a, F(\varepsilon_z))$

Then $\text{Prob}(\underline{a} = \varepsilon_a) = \delta(\varepsilon_a, F(z))$

or succinctly $\underline{a = F(z)}$

1d

So, if f is 1:1

Robustness $\Rightarrow$ Signalling

<u>For dichotomic variables</u>

$$\Delta \text{Prob}(\underline{a} = \varepsilon_a)$$

$$= \left(\text{Prob}(\underline{a} = \varepsilon_a \,/\, \underline{z} = +1) - \text{Prob}(\underline{a} = \varepsilon_a \,/\, \underline{z} = -1) \right) \times \Delta \text{Prob}(\underline{z} = +1)$$